

what's the point of...

POWERS AND ROOTS?

If maths be the root of music, play on

Did you know that musical scales are based on maths? All notes have unique frequencies, with higher notes having higher frequencies, and the relationships between different notes are mathematical.

The simplest of these relationships is the one between the same note in different octaves: to produce the same note one octave higher you double the frequency. For example the note A above middle C has a frequency of 440 Hz; the note A one octave higher has a frequency of 880 Hz. To increase another octave you would double again, so the next note A, an octave higher is at 1760 Hz. You can show this really easily using a guitar: if you pluck the A string, then press down on the twelfth fret and pluck it again you will have halved the length of the vibrating string which doubles the frequency giving you an A one octave higher!

Music based on one note, even played at different octaves, would be very boring. To generate more notes you need to split the octave into a scale. Traditional Western music uses the Chromatic scale. This has as a basis the frequency 440 Hz for the note A (often written A440) and splits the scale into 12 notes. These are the notes you would see on a piano keyboard or guitar fretboard.

The 12 notes the scale is split into are A, A#, B, C, C#, D, D#, E, F, F#, G, G#. (The # symbol means 'sharp'.) The most common way to generate these notes is to use 'equal temperament' where each step up the scale, or semitone, is defined so that the ratio, r , of one note to the previous note is constant. After 12 semitones you should have moved up a complete octave by multiplying by the ratio 12 times. This means that if you start at A440 you get the equation $440 \times r^{12} = 880$ which solves to give

$$r = \sqrt[12]{2} \approx 1.0595$$

This can then be used to generate the frequencies of all the notes in the scale.

Note	Frequency (Hz)
A	440.00
A#	466.16
B	493.88
C#	523.25
C	554.37
D#	587.33
D	622.25
E	659.26
F	698.46
F#	739.99
G	783.99
G#	830.61
A	880.00

To the human ear the increase in the pitch of the notes sounds constant even though the gap between the frequencies is increasing.

Splitting into 12 is not the only way to split an octave. Arabic music uses up to 24 divisions and Chinese music does not use equal temperament but splits an octave so that the ratio of the frequencies of one note to the next is a whole number. This is often referred to as 'just intonation'.

The relationship between maths and music is very rich and this is just one example. There are many other mathematical features that occur in music such as Fibonacci numbers and the golden ratio.

Slide rule

You've probably noticed skid-marks left on roads where cars have had to brake suddenly. Accident investigators can use maths to tell the speed of a car from the length of the skid mark.

The length of the skid mark is proportional to the speed of the car squared. So investigators use the formula: $d = kv^2$ where d is the length of the skid-mark, v is the speed and k is a constant that depends on such things as the road surface and the conditions.

Rearranging this gives:

$$v = \sqrt{\frac{d}{k}}$$

which lets them estimate the speed directly.

There are various reasons why this might be an underestimate of the speed though. Only part of the skid marks may be visible, a collision may have affected them or the brakes may have slowed down the vehicle before the car started skidding. For these reasons the speed calculated is often referred to as the minimum speed, leaving the accident investigators with an inequality, and often more maths to do!



Maths keeps the world in motion

Johannes Kepler lived in central Europe, in what is now part of Germany, between 1571 and 1630. He was an astronomer and mathematician who studied the motion of the planets and was one of the first people to write in defence of Copernicus' model of a sun-centred (or heliocentric) universe; before this most observers believed the Earth was at the centre of the universe. This defence took him eight years to perfect!

Kepler studied data collected from observations by the Danish astronomer Tycho Brahe and suggested three laws for planetary motion. The third of these laws relates the time it takes for a planet to orbit the Sun (its period) with its distance away from the Sun.

Kepler's third law states that the square of the period, P , is proportional to the cube of the distance, d :

$$P^2 \propto d^3 \text{ or } P = kd^{1.5}$$

where k is constant.

Kepler even recorded the day he made the discovery: 15th May 1618, although he did also state that this discovery was a result of seventeen years of hard work – there's a lesson for us all there!

